5–5 Differences of Two Squares

Objective: To simplify products of the form $(a + b)(a - b)$ and to factor differences of two squares.

Vocabulary

Product of the Sum and Difference of Two Numbers
$$(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

Difference of Two Squares
$$a^2 - b^2 = (a + b)(a - b)$$

Example 1 Write each product as a binomial.

a. $(x + 2)(x - 2)$ **b.** $(2n + 3)(2n - 3)$

Solution These products fit the form $(a + b)(a - b)$, so each binomial is of the form $a^2 - b^2$.

a. $(x + 2)(x - 2) = (x)^2 - (2)^2$
$$= x^2 - 4$$

b. $(2n + 3)(2n - 3) = (2n)^2 - (3)^2$
$$= 4n^2 - 9$$

Write each product as a binomial.

1. $(a + 3)(a - 3)$

2. $(4 - x)(4 + x)$

3. $(x + 5)(x - 5)$

4. $(9 - x)(9 + x)$

5. $(5a + 2)(5a - 2)$

6. $(7a - 2)(7a + 2)$

7. $(4 + 3x)(4 - 3x)$

8. $(6 - 5x)(6 + 5x)$

9. $(3 - 5x)(3 + 5x)$

10. $(8x + 7)(8x - 7)$

Example 2 Write each product as a binomial.

a. $(a^2 - 3b)(a^2 + 3b)$ **b.** $(xy + z)(xy - z)$

Solution These products fit the form $(a + b)(a - b)$, so each binomial is of the form $a^2 - b^2$.

a. $(a^2 - 3b)(a^2 + 3b) = (a^2)^2 - (3b)^2$
$$= a^4 - 9b^2$$

b. $(xy + z)(xy - z) = (xy)^2 - z^2$
$$= x^2y^2 - z^2$$

Write each product as a binomial.

11. $(3x + 4y)(3x - 4y)$

12. $(2u + v)(2u - v)$

13. $(x^2 - 8y)(x^2 + 8y)$

14. $(x^2 - 3y^2)(x^2 + 3y^2)$

15. $(2a^2 + 5b^2)(2a^2 - 5b^2)$

16. $(ab - 2c)(ab + 2c)$

5–5 *Differences of Two Squares* (continued)

Example 3 Multiply. Use the pattern $(a + b)(a - b) = a^2 - b^2$.

  a. $58 \cdot 62$ **b.** $93 \cdot 87$

Solution **a.** $58 \cdot 62 = (60 - 2)(60 + 2)$ **b.** $93 \cdot 87 = (90 + 3)(90 - 3)$
                   $= 3600 - 4$ $= 8100 - 9$
                   $= 3596$ $= 8091$

Multiply. Use the pattern $(a + b)(a - b) = a^2 - b^2$.

17. $16 \cdot 24$ **18.** $27 \cdot 33$ **19.** $53 \cdot 47$ **20.** $35 \cdot 45$

21. $41 \cdot 39$ **22.** $92 \cdot 88$ **23.** $104 \cdot 96$ **24.** $60 \cdot 140$

Example 4 Factor: **a.** $a^2 - 16$ **b.** $9 - 4b^2$ **c.** $25a^2 - 36x^6$

Solution Use the pattern $a^2 - b^2 = (a + b)(a - b)$

  a. $a^2 - 16 = a^2 - 4^2$
              $= (a + 4)(a - 4)$

  b. $9 - 4b^2 = 3^2 - (2b)^2$
              $= (3 + 2b)(3 - 2b)$

  c. $25a^2 - 36x^6 = (5a)^2 - (6x^3)^2$
                $= (5a + 6x^3)(5a - 6x^3)$

Factor. You may use a calculator or a table of squares.

25. $b^2 - 16$ **26.** $f^2 - 81$ **27.** $36 - x^2$

28. $9e^2 - 16$ **29.** $49n^2 - 1$ **30.** $4a^2 - 9$

31. $a^4 - 36$ **32.** $49b^2 - 16c^2$ **33.** $100 - 121r^2$

34. $121 - y^2$ **35.** $25u^2 - 36$ **36.** $16x^2 - 225$

Mixed Review Exercises

Simplify.

1. $5z(z - 2) + 3z(z + 4)$ **2.** $(x + 4)(x - 5)$ **3.** $-3(m + 2) - 4m(m - 3)$

4. $\dfrac{36a^5b^2}{9a^3}$ **5.** $\dfrac{15a + 5}{5}$ **6.** $\dfrac{18n^2x}{6nx}$

7. $(a + 2)(2a - 1)$ **8.** $(3b + 2)(b - 1)$ **9.** $(4x)^2\left(\dfrac{1}{4}\right)^2 x$

10. $\dfrac{12y^3 + 28y^2 - 8y}{4y}$ **11.** $\dfrac{30x^3 + 45x^2 - 15x}{15x}$ **12.** $\dfrac{24x^3y^4z}{3x^3y^3z}$